The Switte Programming Language

James Martin jtmar@lijero.co

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1 Purpose

Switte is a general-purpose programming language intended to make traditionally experimental or theoretical programming language concepts more accessible to the everyday programmer.

These are the pragmatic design goals:

- **Accessiblity**: It must be as easy as practical to transition from a "normal" programming language.
- **Interopability**: A programming language is useless if it can't interact with the outside world.
- **Performance**: It must be fast enough for practical use.

These are the idealistic design goals:

- **Minimalism**: The simpler the design, the easier the language is to understand. The simpler the implementation, the better the implementation.

- **Extensibility**: Extensibility means complexity can be moved out of the core into libraries, allowing a much tighter development loop, and specialized features that don't make sense to be built-in.
- **Low-level**: Low-level language cores are simpler to implement and have wider applicability.
- **Theoretic basis**: A theoretic basis makes it easier to reason about the language– for people *and* machines. It also makes for a more powerful language, and potentially applications in research.

Contents

2 Theoretic Background

2.1 Axioms

2.1.1 Axiom of identity

This is the assertion that we can make an assumption. It corresponds with variable introduction in programming.

Given an x, we have an x.

$$\frac{}{x : A \vdash y : A}\ \text{Identity}$$

This can equivalently be written as

Given nothing, we have an x and a demand for an x.

$$\dfrac{\dfrac{}{x : A \vdash y : A}\ \text{Identity}}{\vdash x : A^{\perp}, y : A}\ \text{Left duality introduction}$$

or

Given an x and a demand for an x, we have nothing.

$$\dfrac{\dfrac{}{x : A \vdash y : A}\ \text{Identity}}{x : A, y : A^{\perp} \vdash}\ \text{Right duality elimination}$$

by duality.

2.2 Structural Rules

Structural rules have to do with rearranging contexts, rather than with individual connectives.

2.2.1 Cut Rule

The cut rule has to do with the composition of proofs.

$$\dfrac{\Gamma, x : A \vdash \Theta \qquad \Delta \vdash x : A, \Lambda}{\Gamma, \Delta \vdash \Theta, \Lambda}\ \text{Cut}$$

2.3 Computational Rules

Computational rules have to do with the introduction and elimination of connectives.

2.3.1 Additive Conjunction

Additive conjunction corresponds with conjunction in logic. It is defined in terms of conjunction elimination: $a \wedge b \implies a$ and $a \wedge b \implies b$. Conversely, if $c \implies a$ and $c \implies b$, then $c \implies a \wedge b$.

In terms of resources, $a \wedge b$ is a choice between a and b. You can choose to get an a or a b, but not both. Consider a customer at a pizza parlor: for 20 dollars they can order a cheese pizza or a pepperoni pizza, but they can't get both pizzas from the same 20 dollars.

$$\frac{\Gamma \vdash \Theta, x : A, \Lambda \qquad \Gamma \vdash \Theta, x : B, \Lambda}{\Gamma \vdash \Theta, x : A \wedge B, \Lambda} \wedge \text{ introduction}$$

$$\frac{\Gamma \vdash \Theta, x : A \wedge B, \Lambda}{\Gamma \vdash \Theta, x : A, \Lambda} \wedge \text{ select left}$$

$$\frac{\Gamma \vdash \Theta, x : A \wedge B, \Lambda}{\Gamma \vdash \Theta, x : B, \Lambda} \wedge \text{ select right}$$

$\top$ is the assertion that there exists some inhabited type a. It could be any type, though: there just must be something true. It corresponds with truth in logic. $\top : \exists a.a$, "there exists some inhabited type a".

$\top$ is also the unit for additive conjunction. Given $x \wedge \top$, one must always pick x, because there is no elimination rule for $\top$, and therefore it's impossible to dispose of.

$$\frac{}{\Gamma \vdash \Theta, \top, \Lambda} \top \text{ introduction}$$

2.3.2 Additive Disjunction

Additive disjunction corresponds with disjunction in logic. It is defined in terms of disjunction elimination: if $a \implies c$ and $b \implies c$, then $a + b \implies c$. However, unlike with normal logic, disjunctive syllogism does not apply.

In terms of resources, it is like a choice made by someone else: a pizza parlor might have to make cheese pizzas or pepperoni pizzas, but the customer gets to decide which they have to make.

Given an x, we have an x or a y.

$$\frac{\Gamma \vdash \Theta, x : A, \Lambda}{\Gamma \vdash \Theta, R(x) : A + 0, \Lambda} + \text{ select left}$$

Given a y, we have an x or a y.

$$\frac{\Gamma \vdash \Delta, x : A, \Theta}{\Gamma \vdash \Theta, L(x) : 0 + A, \Lambda} + \text{ select right}$$

If we can prove z from x or we can prove z from y, then we can prove z from x or y.

$$\frac{\Gamma, x : A, \Delta \vdash \Theta \qquad \Gamma, x : B, \Delta \vdash \Theta}{\Gamma, x : A + B, \Delta \vdash \Theta} + \text{ elimination}$$

The unit for additive disjunction is 0. It corresponds with a product that cannot be produced, therefore given $a + 0$, a must always have been selected. Since 0 is impossible, $a + 0$ is just as good as a. This is encoded as 0 being equivalent to $\forall b.b$, which essentially means it entails a, and therefore the same proof we used for a will also work in the case of 0. Logically, it corresponds with falsity.

$$\frac{}{\Gamma, x : 0, \Delta \vdash \Theta} \text{ Explosion}$$

2.3.3 Multiplicative Conjunction

Multiplicative conjunction is pretty straightforward. It's having two things at once. This should require no explanation.

If I can prove x and I can prove y, I can prove x and y.

$$\frac{\Gamma \vdash \Theta, x : A \qquad \Delta \vdash y : B, \Lambda}{\Gamma, \Delta \vdash \Theta, (x, y) : A * B, \Lambda} * \text{ introduction}$$

If given x and given y I can prove z, then given x and y I can prove z.

$$\frac{\Gamma, x : A, y : B, \Delta \vdash \Theta}{\Gamma, (x, y) : A * B, \Delta \vdash \Theta} * \text{ elimination}$$

1 is the unit for multiplicative conjunction. Whereas $\top$ says "there exists an inhabited type", 1 is an example of an inhabited type. In fact, it's definition is basically “this is an inhabited type carrying no information”.

I can prove 1.

$$\frac{\Gamma, \Delta \vdash \Theta}{\Gamma, () : 1, \Delta \vdash \Theta} \text{ 1 introduction}$$

1 proves nothing.

$$\frac{\Gamma, () : 1, \Delta \vdash \Theta}{\Gamma, \Delta \vdash \Theta} \text{ 1 elimination}$$

2.3.4 Multiplicative Disjunction

Multiplicative disjunction is entirely confusing to most people (including me). It means having both an a and a b, but not at the same time. Whereas $a + b$ is defined as “having an a or b”, $a \vee b$ is defined as $a^{\perp} \implies b$ (not a implies b), which is to say disjunctive syllogism.

If one of a or b must be true, then $a \vee b$ must be true.

$$\frac{\Gamma \vdash \Theta, a : X, b : Y, \Lambda}{\Gamma \vdash \Theta, a \vee b : X \vee Y, \Lambda} \vee \text{ introduction}$$

If a implies c and b implies d, then one of a and b implies one of c or d.

$$\frac{\Gamma, x : A \vdash \Delta \qquad y : B, \Theta \vdash \Lambda}{\Gamma, x \vee y : A \vee B, \Theta \vdash \Delta, \Lambda} \vee \text{ elimination}$$

If a implies b, then a implies one of b or falsity.

$$\frac{\Gamma \vdash \Delta, \Theta}{\Gamma \vdash \Delta, \perp, \Theta} \perp \text{ introduction}$$

Falsity implies nothing.

$$\frac{\Gamma, \perp, \Delta \vdash \Theta}{\Gamma, \Delta \vdash \Theta} \perp \text{ elimination}$$

2.4 Duality

Duality corresponds with a demand for a resource, and logical negation. By duality, the identity axiom is the same as the law of the excluded middle.

2.4.1 Duality introduction

$$\frac{\Gamma, x : A \vdash \Theta}{\Gamma \vdash \Theta, x : A^{\perp}} \text{ Duality left introduction}$$

$$\frac{x : A, \Delta \vdash \Lambda}{\Gamma \vdash x : A^{\perp}, \Lambda} \text{ Duality right introduction}$$

$$\frac{\Gamma \vdash \Theta, x : A}{\Gamma, x : A^{\perp} \vdash \Theta} \text{ Duality right elimination}$$

$$\frac{\Delta \vdash x : A, \Lambda}{x : A^{\perp}, \Delta \vdash \Lambda} \text{ Duality left elimination}$$

2.4.2 Duality definition

A demand for a demand is the same as having (double negation elimination): $(A^{\perp})^{\perp} \equiv x : A$

A demand for an option is the same as either one demand or the other: $(A \wedge B)^{\perp} \equiv A^{\perp} + B^{\perp}$

A demand for either one or the other is the same as an option between demands: $(A + B)^{\perp} \equiv A^{\perp} \wedge B^{\perp}$

A demand for two things is the same as preparing to satisfy two demands: $(A * B)^\perp \equiv A^\perp \vee B^\perp$

A demand for a preparation for two things is the same as two demands: $(A \vee B)^\perp \equiv A^\perp * B^\perp$

A demand for unsasisfiability is impossible: $\top^\perp \equiv 0$

A demand for the impossible is unsatisfiable: $0 \equiv \top^\perp$

A demand for truth is ignorable: $1^\perp \equiv \perp$

A demand for nothing is truth: $\perp \equiv 1^\perp$

2.5 Implication

Implication can be defined in terms of the existing rules: $x \impliedby y \equiv x \vee y^\perp$ and $x \implies y \equiv x^\perp \vee y$.

You can define these pseudo-rules:

$$\frac{\Gamma, x : A \vdash \Theta}{\Gamma \vdash \Theta \impliedby x : A} \text{ Left implication introduction}$$

$$\frac{\Gamma \vdash \Theta \impliedby x : A}{\Gamma, x : A \vdash \Theta} \text{ Left implication elimination}$$

and the same on the other side.

2.6 Paraconsistency

The principle of explosion requires disjunction introduction followed by disjunctive syllogism. However, disjunction introduction is only applicable to $+$, and disjunctive syllogism is only applicative to $\vee$. Therefore, this logic is paraconsistent.